How to Factor

Hopefully you know how to factor. If not here are some helpful hints on factoring.

1. **Take a Common Factor**
2. **Look for Special Cases**
3. **If in the form** $x^2+bx+c=0$ **(Factor Directly using Sum and Difference)**
4. **If in the form** $ax^2+bx+c=0$

Then Guess and Check, or use Advance Sum and Difference, or Split the Middle Term, or Complete the Square, or use the Quadratic Formula.

5. **If there are more than 3 terms try grouping terms**

If 4 terms try to group 2 sets of 2 terms each that have a common factor.
If 6 terms try to group 2 sets of 3 terms with common factors or 3 sets of 2 terms with a common factor.

6. **Look for Perfect Squares.**
7. **If this does not work wait for Factor Theorem, and Extended Factor Theorem in Chapter 2.**

How to Factor

Factoring is fairly easy if you follow the following steps. Do the following steps in order.

1. Take a Common Factor

When you are factoring the FIRST thing is to take out all common factors. Taking out a common factor simplifies future factoring tasks.

Example: $y=4x+8$

$y=4(x+2)$

2. Look for Special Cases

Pay Special Attention to special cases. Harcourt uses special cases all over this book. Difference of Squares is the most common Special Case, but Perfect Squares are also very common.

Difference of Squares

$a^2-b^2=(a)^2-(b)^2=(a-b)(a+b)$

Difference of Squares works when there is only 2 terms separated by a "-" term. "a", and "b" are the square roots of the first term, and second term.

Example 1

$x^2-9=(x)^2-(3)^2=(x-3)(x+3)$

$a=\sqrt{x^2}=x$ and $b=\sqrt{9}=3$

Example 2

$4x^2-81y^4z^6=(2x)^2-(9y^2z^3)^2$

$=(2x-9y^2z^3)(2x+9y^2z^3)$

$a=\sqrt{4x^2}=2x$ and $b=\sqrt{81y^4z^6}=9y^2z^3$

Perfect Squares

$a^2+2ab+b^2=(a+b)(a+b)=(a+b)^2$

or

$a^2-2ab+b^2=(a-b)(a-b)=(a-b)^2$

This works when the second term is

$\pm2\sqrt{1^{st}\,term}\sqrt{3^{rd}\,term}=\pm2\sqrt{a^2}\sqrt{b^2}$

Example 1

$x^2+6x+9=(x)^2+2(x)(3)+(3)^2=(x+3)^2$

Note the second term $2\sqrt{x^2}\sqrt{9}=6x$

Example 2

$4x^2+12x+9=(2x)^2+2(2x)(3)+(3)^2=(2x+3)^2$

Note the second term $2\sqrt{4x^2}\sqrt{9}=12x$

3. If in the form $x^2+bx+c=0$ (Factor Directly)

This only works if 2 numbers exist that add to the middle term and multiply to the third term. If not use Complete the Square or Quadratic Equation.

$y=x^2+bx+c$
$=x^2+x(d+e)+(d\times e)$
$=(x+d)(x+e)$

Look for 2 Numbers that Add to b. $b=d+e$
Look for 2 Numbers that Multiply to c. $c=d\times e$

Example 1

$y=x^2+5x+4$
$\therefore y=(x+1)(x+4)$

Look for 2 Numbers that Add to 5.
Look for 2 Numbers that Multiply to 4. $1\times4=4$
$2\times2=4$
Note: $5=1+4$ So the 2 numbers are +1 and +4 this time.

Example 2

$y=x^2-5x+4$
$\therefore y=(x-1)(x-4)$

Look for 2 Numbers that Add to -5 .
Look for 2 Numbers that Multiply to 4. $1\times4=4$
$2\times2=4$
Note: $-5=-1-4$ So the 2 numbers are -1 and -4 this time.

Example 3

$y=x^2-3x-4$

$\therefore y=(x+1)(x-4)$

Look for 2 Numbers that Add to -3.
Look for 2 Numbers that Multiply to -4. $1\times4=4$
$2\times2=4$
Note: $-3=1-4$ So the 2 numbers are +1 and -4 this time.

4. If in the form $ax^2+bx+c=0$ (Note: Remember to look for a common factor.)

These questions can be solved by check and guess, completing the square, splitting the middle term or quadratic formula. Remember to look for a common factor. Taking out all common factors helps to make the question simpler. Use the method you are most comfortable with, but some methods are quicker depending on the question. Use check and guess if the question is simple or you are very good at mental mathematics. Use split the middle term, or complete the square if the question is more complicated.

<u>Guess and Check</u> - Guess and check works best if A and C have Few Factors or you are Great at Mental Math..

$y=ax^2+bx+c=(dx+e)(fx+g)$

$a=d\times f$
$b=d\times g+e\times f$
$c=e\times g$

Example 1

$y=2x^2+3x+1=(2x+1)(x+1)$

$a=2=2\times1$ a is a prime number and only has 2 factors.
$c=1=1\times1$ c is a prime number and only has 1 factor.
Only 1 guess exists $(2x+1)(x+1)$
Check $(2x+1)(x+1)$ using FOIL or $b=d\times g+e\times f$

Advanced Sum and Difference - This works ONLY if 2 numbers ADD to b and MULTIPLY to $(a \times c)$.

$y = ax^2 + bx + c = a\left(x + \frac{d}{a}\right)\left(x + \frac{e}{a}\right)$

Look for 2 numbers that add to b
Look for 2 numbers that multiply to $a \times c$

Example 1

$y = 2x^2 + 3x + 1$

Look for 2 numbers that add to 3
Look for 2 numbers that multiply to $2 \times 1 = 2$
The numbers are 1 and 2.

$y = 2\left(x + \frac{1}{2}\right)(x+1)$

Put the "2" in front, and divide both numbers by 2.
Divide 1 by 2, so 1 becomes "$\frac{1}{2}$". Divide 2 by 2, so 2 becomes "$\frac{2}{2} = 1$".

$y = (2x+1)(x+1)$

Simplify

Example 2

$10m^2 - 3m - 1$

Look for 2 numbers that add to -3
Look for 2 numbers that multiply to $10 \times (-1) = -10$
The numbers are 2 and -5.

$= 10\left(m + \frac{2}{10}\right)\left(m - \frac{5}{10}\right)$

Put the "10" in front, and divide both numbers by 10.

$= 5 \cdot 2 \cdot \left(m + \frac{1}{5}\right)\left(m - \frac{1}{2}\right)$

Divide 2 by 10, so 2 becomes "$\frac{2}{10} = \frac{1}{5}$". Divide -5 by 10, so -5 becomes "$\frac{-5}{10} = \frac{-1}{2}$".

$= 5 \cdot \left(m + \frac{1}{5}\right) \cdot 2 \cdot \left(m - \frac{1}{2}\right)$

$= (5m+1)(2m-1)$

Simplify

Split The Middle Term - This works ONLY if 2 numbers ADD to b and MULTIPLY to $(a \times c)$.

$y = ax^2 + bx + c$

Look for 2 numbers that add to b
Look for 2 numbers that multiply to $a \times c$

$y = 2x^2 + 3x + 1$

Look for 2 numbers that add to 3
Look for 2 numbers that multiply to $2 \times 1 = 2$
The numbers are 1 and 2.

$y = 2x^2 + (1x + 2x) + 1$

$1 \times 2 = 2$ and $1 + 2 = 3$ $\therefore 3x = 1x + 2x$

$y = (2x^2 + 1x) + (2x + 1)$

Group first two and final two terms. Put a "$+$" sign between the groups.

$y = x(2x+1) + 1(2x+1)$

Take common factors from the first set of terms and the second set of terms.

$y = (2x+1)(x+1)$

Take common factor $(2x+1)$ from each set of terms.

<u>Complete The Square</u> – Complete the Square where *x* is an Integer.

$y = 2x^2 + 3x + 1$

$y = 2\left(x^2 + \frac{3}{2}x + \frac{1}{2}\right)$ Take "2" out as a common factor

$y = 2\left[x^2 + 2x\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + \frac{1}{2}\right]$ Complete the square inside the bracket

$y = 2\left[\left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{1}{2}\right]$ Simplify inside the bracket

$y = 2\left[\left(x + \frac{3}{4}\right)^2 - \frac{1}{16}\right]$

$y = 2\left[\left(x + \frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right]$ This will be difference of squares.

$y = 2\left[\left(x + \frac{3}{4}\right) - \left(\frac{1}{4}\right)\right]\left[\left(x + \frac{3}{4}\right) + \left(\frac{1}{4}\right)\right]$

$y = 2\left(x + \frac{3}{4} - \frac{1}{4}\right)\left(x + \frac{3}{4} + \frac{1}{4}\right)$

$y = 2\left(x + \frac{1}{2}\right)(x + 1)$

$y = (2x + 1)(x + 1)$ The answer is the same as guess and check.

<u>Complete The Square</u> – where *x* is NOT an Integer.

$$y = 2x^2 + 2x - 1$$

$$y = 2\left(x^2 + x - \frac{1}{2}\right)$$

$$y = 2\left[x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{1}{2}\right]$$

$$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{2}\right]$$

$$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \frac{3}{4}\right]$$

$$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{\sqrt{4}}\right)^2\right]$$

$$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right]$$

$$y = 2\left[\left(x + \frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\right]\left[\left(x + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\right]$$

$$y = 2\left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}\right)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}\right)$$

$$y = 2\left(x + \frac{1 - \sqrt{3}}{2}\right)\left(x + \frac{1 + \sqrt{3}}{2}\right)$$

<u>Quadratic Formula</u> - where *x* is NOT an Integer.

$y = 2x^2 + 2x - 1 \quad a = 2,\ b = 2,\ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \left(2\sqrt{3}\right)}{2(2)}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{4}$$

$$\sqrt{b^2 - 4ac} = \sqrt{(2)^2 - 4(2)(-1)} = \sqrt{4 + 8} = \sqrt{12} = 2\sqrt{3}$$

$$\therefore x = \frac{-2 + 2\sqrt{3}}{4} \quad \text{or} \quad \therefore x = \frac{-2 - 2\sqrt{3}}{4}$$

$$x = \frac{-1 + \sqrt{3}}{2} \qquad x = \frac{-1 - \sqrt{3}}{2}$$

$$\left(x + \frac{1 - \sqrt{3}}{2}\right) = 0 \qquad \left(x + \frac{1 + \sqrt{3}}{2}\right) = 0$$

$$\therefore y = 2\left(x + \frac{1 - \sqrt{3}}{2}\right)\left(x + \frac{1 + \sqrt{3}}{2}\right)$$

Complete The Square – where *x* is Imaginary.

$y = 2x^2 + 2x + 1$

$y = 2\left(x^2 + x + \frac{1}{2}\right)$

$y = 2\left[x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{1}{2}\right]$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{1}{2}\right]$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}\right]$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{-1}{4}\right)\right]$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{-1}{4}}\right)^2\right]$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{-1}}{2}\right)^2\right]$ $\qquad \sqrt{-1} = i$

$y = 2\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{i}{2}\right)^2\right]$

$y = 2\left[\left(x + \frac{1}{2}\right) - \left(\frac{i}{2}\right)\right]\left[\left(x + \frac{1}{2}\right) + \left(\frac{i}{2}\right)\right]$

$y = 2\left(x + \frac{1}{2} - \frac{i}{2}\right)\left(x + \frac{1}{2} + \frac{i}{2}\right)$

$y = 2\left(x + \frac{1-i}{2}\right)\left(x + \frac{1+i}{2}\right)$

Quadratic Formula - where *x* is Imaginary.

$y = 2x^2 + 2x + 1 \qquad a = 2,\ b = 2,\ c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sqrt{b^2 - 4ac}$
$= \sqrt{(2)^2 - 4(2)(1)}$
$= \sqrt{4 - 8}$
$= \sqrt{-4} = 2i$

$x = \frac{-(2) \pm (2i)}{2(2)}$

$x = \frac{-2 \pm 2i}{4}$

$\therefore x = \frac{-2 + 2i}{4}$ or $x = \frac{-2 - 2i}{4}$

$\therefore x = \frac{-1 + i}{2}$ $\qquad x = \frac{-1 - i}{2}$

$\left(x + \frac{1-i}{2}\right) = 0$ $\qquad \left(x + \frac{1+i}{2}\right) = 0$

$\therefore y = 2\left(x + \frac{1-i}{2}\right)\left(x + \frac{1+i}{2}\right)$

Quadratic Formula - where *x* is an Integer.

$y = 2x^2 + 3x + 1$ — Write question $a = 2$, $b = 3$, $c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Write Quadratic Formula.

$\sqrt{b^2 - 4ac}$ — Write Discriminate Formula $\sqrt{b^2 - 4ac}$

$= \sqrt{(3)^2 - 4(2)(1)}$

$= \sqrt{9 - 8}$

$= \sqrt{1} = 1$ — Determine the value of the Discriminate.

$x = \frac{-(3) \pm (1)}{2(2)}$ — Substitute the values for a, b, and $\sqrt{b^2 - 4ac}$ into the Quadratic Equation.

$x = \frac{-3 \pm 1}{4}$ — Simplify the resulting equation.

$\therefore x = \frac{-3+1}{4}$ or $x = \frac{-3-1}{4}$ — Split simplified equation into 2 equations.

$x = -\frac{1}{2}$ $\quad$ $x = -1$ — Solve for x

$\left(x + \frac{1}{2}\right) = 0$ $\quad$ $(x+1) = 0$ — Set equal to 0 to create factors.

$\therefore y = 2\left(x + \frac{1}{2}\right)(x+1)$ — Substitute back into the general formula $y = a(1^{st}\ factor)(2^{nd}\ factor)$

$\therefore y = (2x+1)(x+1)$ — Simplify to create final factor form. Check to determine if any more factors can be removed.

Quadratic Formula - where *x* is not an Integer.

$y = 2x^2 + 2x - 1$ — Write question $a = 2$, $b = 2$, $c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Write Quadratic Formula.

$\sqrt{b^2 - 4ac}$ — Write Discriminate Formula $\sqrt{b^2 - 4ac}$

$= \sqrt{(2)^2 - 4(2)(-1)}$

$= \sqrt{4 + 8}$

$= \sqrt{12} = 2\sqrt{3}$ — Determine the value of the Discriminate.

$x = \frac{-(2) \pm \left(2\sqrt{3}\right)}{2(2)}$ — Substitute the values for a, b, and $\sqrt{b^2 - 4ac}$ into the Quadratic Equation.

$x = \left(\frac{2}{2}\right)\frac{-1 \pm \sqrt{3}}{2}$ — Simplify the resulting equation.

$x = \frac{-1 \pm \sqrt{3}}{2}$

$\therefore x = \frac{-1+\sqrt{3}}{2}$ or $\therefore x = \frac{-1-\sqrt{3}}{2}$ — Split simplified equation into 2 equations.

$\left(x - \frac{-1+\sqrt{3}}{2}\right) = 0$ $\quad$ $\left(x - \frac{-1-\sqrt{3}}{2}\right) = 0$ — Set equal to 0 to create factors.

$\therefore y = 2\left(x - \frac{-1+\sqrt{3}}{2}\right)\left(x - \frac{-1-\sqrt{3}}{2}\right)$ — Substitute back into the general formula $y = a(1^{st}\ factor)(2^{nd}\ factor)$

<u>Quadratic Formula</u> - where *x* is Imaginary.

$y = 2x^2 + 2x + 1$		Write question $a = 2$, $b = 2$, $c = 1$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\sqrt{b^2 - 4ac}$	- Write Quadratic Formula.
	$= \sqrt{(2)^2 - 4(2)(1)}$	- Write Discriminate Formula $\sqrt{b^2 - 4ac}$
	$= \sqrt{4-8}$	- Determine the value of the Discriminate.
$x = \frac{-(2) \pm (2i)}{2(2)}$	$= \sqrt{-4} = 2i$	- Substitute the values for a, b, and $\sqrt{b^2 - 4ac}$ into the Quadratic Equation.
$x = \left(\frac{2}{2}\right)\frac{-1 \pm i}{2}$		- Simplify the resulting equation.
$x = \frac{-1 \pm i}{2}$		
$\therefore x = \frac{-1+i}{2}$	or $\therefore x = \frac{-1-i}{2}$	- Split simplified equation into 2 equations.
$\left(x - \frac{-1+\sqrt{3}}{2}\right) = 0$	$\left(x - \frac{-1-\sqrt{3}}{2}\right) = 0$	- Set equal to 0 to create factors.
$\therefore y = 2\left(x - \frac{-1+\sqrt{3}}{2}\right)\left(x - \frac{-1-\sqrt{3}}{2}\right)$		- Substitute back into the general formula $y = a\left(1^{st}\ factor\right)\left(2^{nd}\ factor\right)$

5. <u>If there are more than 3 terms.</u>

Remember to always take out a common factor. Taking out a common factor always makes the question simpler.

If 4 terms try to group 2 sets of 2 terms each that have a common factor.
If 6 terms try to group 2 sets of 3 terms with common factors or 3 sets of 2 terms with a common factor.

$kx + px - ky - py$	If there are 4 terms, group 2 sets of 2 terms with common factors.
$= (kx + px) + (-ky - py)$	It is easiest if each set of terms is in brackets separated by an addition sign.
$= x(k + p) + (-y)(k + p)$	Take a common factor from each set of terms. Please note that each set of terms must have a common factor.
$= (x - y)(k + p)$	Take a common factor from each set of terms.

6. <u>Look for Perfect Squares.</u>

$4y^2 + 4yz + z^2 - 1$	Note the y^2, z^2, and yz term. This means it might be possible to complete the square.
$= \left(4y^2 + 4yz + z^2\right) - (1)^2$	
$= \left[(2y)^2 + 2(y)(z) + (z)^2\right] - (1)^2$	
$= (2y + z)^2 - (1)^2$	This is difference of squares.
$= \left[(2y + z) - 1\right]\left[(2y + z) + 1\right]$	
$= (2y + z - 1)(2y + z + 1)$	

Review of Prerequisite Skills - Answers

1 a. $p^2+2pr+r^2$
$=(p)^2+2(p)(q)+(r)^2$
$=(p+r)^2$

b. $16n^2+8n+1$
$=(4n)^2+2(4n)(1)+(1)^2$
$=(4n+1)^2$

Or

$16n^2+8n+1$
$=(16n^2+4n)+(4n+1)$
$=4n(4n+1)+1(4n+1)$
$=(4n+1)(4n+1)$
$=(4n+1)^2$

c. $9u^2+30u+25$
$=(3u)^2+2(3u)(5)+(5)^2$
$=(3u+5)^2$

Or

$9u^2+30u+25$
$=(9u^2+15u)+(15u+25)$
$=3u(3u+5)+5(3u+5)$
$=(3u+5)(3u+5)$
$=(3u+5)^2$

d. v^2+4v+3
$=(v+1)(v+3)$

e. $2w^2+3w+1$
$=(2w^2+1w)+(2w+1)$
$=w(2w+1)+1(2w+1)$
$=(w+1)(2w+1)$

f. $3k^2+7k+2$
$=(3k^2+6k)+(1k+2)$
$=3k(k+2)+1(1k+2)$
$=(3k+1)(k+2)$

g. $7y^2+15y+2$
$=(7y^2+1y)+(14y+2)$
$=y(7y+1)+2(7y+1)$
$=(7y+1)(y+2)$

h. $5x^2-16x+3$
$=(5x^2-1x)+(-15x+3)$
$=x(5x-1)+(-3)(5x-1)$
$=(x-3)(5x-1)$

i. Book says $3v^2-11v-10$ (Does not Factor)
Book should say
$3v^2-11v+10$
$=(3v^2-5v)+(-6v+10)$
$=v(3v-5)+(-2)(3v-5)$
$=(v-2)(3v-5)$

2 a. $25x^2-y^2$
$=(5x)^2-(y)^2$
$=(5x-y)(5x+y)$

b. m^2-p^2
$=(m)^2-(p)^2$
$=(m-p)(m+p)$

c. $1-16r^2$
$=(1)^2-(4r)^2$
$=(1-4r)(1+4r)$

d. $49m^2-64$
$=(7m)^2-(8)^2$
$=(7m-8)(7m+8)$

e. $p^2r^2-100x^2$
$=(pr)^2-(10x)^2$
$=(pr-10x)(pr+10x)$

f. $3-48y^2$
$=3(1-16y^2)$
$=3\left[(1)^2-(4y)^2\right]$
$=3(1-4y)(1+4y)$

g. $(x+n)^2-9$
$=(x+n)^2-(3)^2$
$=\left[(x+n)-3\right]\left[(x+n)+3\right]$
$=(x+n-3)(x+n+3)$

h. $49u^2-(x-y)^2$
$=(7u)^2-(x-y)^2$
$=[7u-(x-y)][7u+(x-y)]$
$=(7u-x+y)(7u+x-y)$

i. x^4-16
$=(x^2)^2-(4)^2$
$=(x^2-4)(x^2+4)$
$=[(x)^2-(2)^2](x^2+4)$
$=(x-2)(x+2)(x^2+4)$

3. a. $kx+px-ky-py$
$=(kx+px)+(-ky-py)$
$=x(k+p)+(-y)(k+p)$
$=(x-y)(k+p)$

b. $fx-gy+gx-fy$
$=(fx+gx)+(-fy-gy)$
$=x(f+g)+(-y)(f+g)$
$=(x-y)(f+g)$

c. h^3+h^2+h+1
$=(h^3+h^2)+(h+1)$
$=h^2(h+1)+1(h+1)$
$=(h^2+1)(h+1)$

d. $x-d+(x-d)^2$
$=(x-d)+(x-d)^2$
$=(x-d)1+(x-d)(x-d)$
$=(x-d)(1+x-d)$
$=(x-d)(x-d+1)$

e. $4y^2+4yz+z^2-1$
$=(4y^2+4yz+z^2)-(1)^2$
$=[(2y)^2+2(y)(z)+(z)^2]-(1)^2$
$=(2y+z)^2-(1)^2$
$=[(2y+z)-1][(2y+z)+1]$
$=(2y+z-1)(2y+z+1)$

f. $x^2-y^2+z^2-2xz$
$=(x^2-2xz+z^2)-(y^2)$
$=[(x)^2+2(x)(-z)+(-z)^2]-(y)^2$
$=(x-z)^2-(y)^2$
$=[(x-z)-y][(x-z)+y]$
$=(x-z-y)(x-z+y)$

4. a. $4x^2+2x-6$
$=2(2x^2+x-3)$
$=2[(2x^2+3x)+(-2x-3)]$
$=2[x(2x+3)+(-1)(2x+3)]$
$=2[(2x+3)(x-1)]$
$=2(2x+3)(x-1)$

b. Book says $28s^2+8st+20t^2$
$28s^2+8st+20t^2$
$=4(7s^2+2st+5t^2)$

Book should say
$28s^2+8st-20t^2$
$=4(7s^2+2st-5t^2)$
$=4[(7s^2+7st)+(-5st-5t^2)]$
$=4[7s(s+t)+(-5t)(s+t)]$
$=4(7s-5t)(s+t)$

c. $y^2-(r-n)^2$
$=(y)^2-(r-n)^2$
$=[y-(r-n)][y+(r-n)]$
$=(y-r+n)(y+r-n)$

d. $8+24m-80m^2$
$=-80m^2+24m+8$
$=(-8)(10m^2-3m-1)$
$=(-8)[(10m^2-5m)+(2m-1)]$
$=-8[5m(2m-1)+1(2m-1)]$
$=-8[(5m+1)(2m-1)]$
$=-8(5m+1)(2m-1)$
$=8(5m+1)(-1)(2m-1)$
$=8(5m+1)(1-2m)$

e. $6x^2-13x+6$
$=(6x^2-9x)+(-4x+6)$

$= 3x(2x-3)+(-2)(2x-3)$
$= (2x-3)(3x-2)$

f. y^3+y^2-5y-5
$= (y^3+y^2)+(-5y-5)$
$= y^2(y+1)+(-5)(y+1)$
$= (y^2-5)(y+1)$
$= (y-\sqrt{5})(y+\sqrt{5})(y+1)$

g. $60y^2-10y-120$
$= 10(6y^2-y-12)$
$= 10[(6y^2-9y)+(8y-12)]$
$= 10[3y(2y-3)+4(2y-3)]$
$= 10[(3y+4)(2y-3)]$
$= 10(3y+4)(2y-3)$

h. $10x^2+38x+20$
$= 2(5x^2+19x+10)$

i $27x^2-48$
$= 3(9x^2-16)$
$= 3[(3x)^2-(4)^2]$
$= 3(3x-4)(3x+4)$

5 a $36(2x-y)^2-25(u-2y)^2$
$= [6(2x-y)]^2-[5(u-2y)]^2$
$= [6(2x-y)-5(u-2y)][6(2x-y)+5(u-2y)]$
$= (12x-6y-5u+10y)(12x-6y+5u-10y)$
$= (12x+4y-5u)(12x-16y+5u)$

b. $g(1-x)-gx+gx^2$
$= g(1-x)+(-gx+gx^2)$
$= g(1-x)+(-gx)(1-x)$
$= (1-x)(g-gx)$
$= g(1-x)(1-x)$
$= g(1-x)^2$

The book said answer is $g(1-x)(1+x)$

Or

The book said $g(1-x)-gx+gx^2$
The book wanted to say

$g(1-x)+gx-gx^2$
$= g(1-x)+(gx-gx^2)$
$= g(1-x)+gx(1-x)$
$= (1-x)(g+gx)$
$= g(1-x)(1+x)$

c Group 3 sets of 2 terms
$y^5-y^4+y^3-y^2+y-1$
$= (y^5-y^4)+(y^3-y^2)+(y-1)$
$= y^4(y-1)+y^2(y-1)+1(y-1)$
$= (y-1)(y^4+y^2+1)$
$= (y-1)[y^4+(2y^2-y^2)+1]$
$= (y-1)[(y^4+2y^2+1)-(y)^2]$
$= (y-1)[(y^2+1)^2-(y)^2]$
$= (y-1)[(y^2+1)+(y)][(y^2+1)-(y)]$
$= (y-1)(y^2+y+1)(y^2-y+1)$

Or Group 2 sets of 3
$y^5-y^4+y^3-y^2+y-1$
$= y^5+y^3+y-y^4+y^2-1$
$= (y^5+y^3+y)+(-y^4-y^2-1)$
$= y(y^4+y^2+1)+(-1)(y^4+y^2+1)$
$= (y-1)(y^4+y^2+1)$
$= (y-1)[y^4+(2y^2-y^2)+1]$
$= (y-1)[(y^4+2y^2+1)-(y)^2]$
$= (y-1)[(y^2+1)^2-(y)^2]$
$= (y-1)[(y^2+1)+(y)][(y^2+1)-(y)]$
$= (y-1)(y^2+y+1)(y^2-y+1)$

Or Group 2 sets of 3
$y^5-y^4+y^3-y^2+y-1$
$= (y^5-y^4+y^3)+(-y^2+y-1)$
$= y^3(y^2-y+1)+(-1)(y^2-y+1)$
$= (y^3-1)(y^2-y+1)$ (see difference of cubes)
$= (y-1)(y^2+y+1)(y^2-y+1)$

d $n^4+2n^2w^2+w^4$
$= (n^2)^2+2(n^2)(w^2)+(w^2)^2$

$=\left(n^2+w^2\right)^2$

e. $9(x+2y+z)^2-16(x-2y+z)^2$

$=\left[3(x+2y+z)\right]^2-\left[4(x-2y+z)\right]^2$

$=(3x+6y+3z)^2-(4x-8y+4z)^2$

$=\left[(3x+6y+3z)-(4x-8y+4z)\right]$

$\times\left[(3x+6y+3z)+(4x-8y+4z)\right]$

$=(3x-4x+6y+8y+3z-4z)$

$\times(3x+4x+6y-8y+3z+4z)$

$=(-x+14y-z)(7x-2y+7z)$

f. $8u^2(u+1)+2u(u+1)-3(u+1)$

$=(u+1)\left(8u^2+2u-3\right)$

$=(u+1)\left[\left(8u^2+6u\right)+(-4u-3)\right]$

$=(u+1)\left[2u(4u+3)+(-1)(4u+3)\right]$

$=(u+1)(4u+3)(2u-1)$

g. $9y^4+12y^2+4$

$=9\left(y^2\right)^2+12\left(y^2\right)+4$

$=\left(3y^2\right)^2+2\left(y^2\right)(6)+(2)^2$

$=\left(3y^2+2\right)^2$

i $abx^2+(an+bm)x+mn$

$=(ax+m)(bx+n)$

or

$=\left(abx^2+anx\right)+(bmx+mn)$

$=ax(bx+n)+m(bx+n)$

$=(ax+m)(bx+n)$

j $x^2+2+\dfrac{1}{x^2}$

$=(x)^2+2x\left(\dfrac{1}{x}\right)+\left(\dfrac{1}{x}\right)^2$

$=\left(x+\dfrac{1}{x}\right)^2$

or

$=\dfrac{1}{x^2}\left(x^4+2x^2+1\right)$

$=\dfrac{1}{x^2}\left[\left(x^2\right)^2+2\left(x^2\right)(1)+(1)^2\right]$

$=\dfrac{1}{x^2}\left(x^2+1\right)\left(x^2+1\right)$

$=\dfrac{1}{x}\left(x^2+1\right)\dfrac{1}{x}\left(x^2+1\right)$

$=\left(x+\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}\right)$

$=\left(x+\dfrac{1}{x}\right)^2$