Complete the Square

The goal of completing the square is to turn a Quadratic Equation into Vertex Form using the perfect square.

Quadratic Equation

$y = x^2 + bx + c$ or $y = ax^2 + bx + c$

Vertex Form

$y = (x-h)^2 + k$ or $y = a(x-h)^2 + k$

In both cases the Vertex is (h,k).

Perfect Square

$$(a+b)^2 = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Many students think completing the square is difficult. Completing the square is easy because it is the same method for **EVERY SINGLE TIME**. If you can learn the method you will get every question perfect.

Complete the Square $y = (x-h)^2 + k$ the Vertex is (h,k)

$y = x^2 + 4x + 9$ - Quadratic Equation

$= (x^2 + 4x) + 9$ - Group First 2 terms

$= \left[(x)^2 + 2x(+2)\right] + 9$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x(+2) + (+2)^2}_{\downarrow} \underbrace{- (+2)^2}_{\downarrow}\right] + 9$ - Complete the Square by adding and subtracting $(+2)^2$

$= \left[\quad (x+2)^2 \quad - \quad 4 \quad\right] + 9$ - Make the Perfect Square $(x)^2 + 2x(+2) + (+2)^2 = (x+2)^2$

$= (x+2)^2 - 4 + 9$ - Remove "[ ]" Brackets

$= (x+2)^2 + 5$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-2,5)$

Example 1 - The red template is the **same for EVERY QUESTION!!!**

- **Blue** is the same everywhere in the question.

$y = x^2 + 4x + 9$ - Quadratic Equation

$= (x^2 + 4x) + 9$ - Group First 2 terms

$= \left[(x)^2 + 2x(+2)\right] + 9$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x(+2) + (+2)^2}_{\downarrow} \underbrace{- (+2)^2}_{\downarrow}\right] + 9$ - Complete the Square by adding and subtracting $(+2)^2$

$= \left[\quad (x+2)^2 \quad - \quad 4 \quad\right] + 9$ - Make the Perfect Square $(x)^2 + 2x(+2) + (+2)^2 = (x+2)^2$

$= (x+2)^2 - 4 + 9$ - Remove "[ ]" Brackets

$= (x+2)^2 + 5$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-2,5)$

- The Blue numbers are the same everywhere in a question.

- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 2
- The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = x^2 + 6x + 7$ - Quadratic Equation

$= \left(x^2 + 6x\right) + 7$ - Group First 2 terms

$= \left[(x)^2 + 2x(+3)\right] + 7$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x(+3) + (+3)^2}_{\downarrow} \underbrace{- (+3)^2}_{\downarrow}\right] + 7$ - Complete the Square by adding and subtracting $(+3)^2$

$= \left[\quad (x+3)^2 \quad - \quad 9 \quad\right] + 7$ - Make the Perfect Square $(x)^2 + 2x(+3) + (+3)^2 = (x+3)^2$

$= (x+3)^2 - 9 + 7$ - Remove "[ ]" Brackets

$= (x+3)^2 - 2$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-3, -2)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 3
- The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = x^2 - 12x + 7$ - Quadratic Equation

$= \left(x^2 - 12x\right) + 7$ - Group First 2 terms

$= \left[(x)^2 + 2x(-6)\right] + 7$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x(-6) + (-6)^2}_{\downarrow} \underbrace{- (-6)^2}_{\downarrow}\right] + 7$ - Complete the Square by adding and subtracting $(-6)^2$

$= \left[\quad (x-6)^2 \quad - \quad 36 \quad\right] + 7$ - Make the Perfect Square $(x)^2 + 2x(-6) + (-6)^2 = (x-6)^2$

$= (x-6)^2 - 36 + 7$ - Remove "[ ]" Brackets

$= (x-6)^2 - 29$ - Now you have Vertex Form

$\therefore$ The Vertex is $(+6, -29)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 4 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = x^2 - 2x + 5$ - Quadratic Equation

$= (x^2 - 2x) + 5$ - Group First 2 terms

$= \left[(x)^2 + 2x(-1)\right] + 5$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x(-1) + (-1)^2}_{\downarrow} \underbrace{- (-1)^2}_{\downarrow}\right] + 5$ - Complete the Square by adding and subtracting $(-1)^2$

$= \left[\quad (x-1)^2 \quad - \quad 1 \quad\right] + 5$ - Make the Perfect Square $(x)^2 + 2x(-1) + (-1)^2 = (x-1)^2$

$= (x-1)^2 - 1 + 5$ - Remove "[ ]" Brackets

$= (x-1)^2 + 4$ - Now you have Vertex Form

$\therefore$ The Vertex is $(+1, 4)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 5 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = x^2 + 5x - 3$ - Quadratic Equation

$= (x^2 + 5x) - 3$ - Group First 2 terms

$= \left[(x)^2 + 2x\left(+\frac{5}{2}\right)\right] - 3$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2}_{\downarrow} \underbrace{- \left(+\frac{5}{2}\right)^2}_{\downarrow}\right] - 3$ - Complete the Square by adding and subtracting $\left(+\frac{5}{2}\right)^2$

$= \left[\quad \left(x + \frac{5}{2}\right)^2 \quad - \quad \frac{25}{4} \quad\right] - 3$ - Make the Perfect Square $(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2$

$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 3$ - Remove "[ ]" Brackets

$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{3}{1} \times \frac{4}{4}$ - Make a Common Denominator

$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{12}{4}$ - Simplify

$= \left(x + \frac{5}{2}\right)^2 - \frac{37}{4}$ - Now you have Vertex Form

$\therefore$ The Vertex is $\left(-\frac{5}{2}, -\frac{37}{4}\right)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 6 - The red template is the **same for EVERY QUESTION!!!**

- Blue is the same everywhere in the question.

$y = x^2 - x - 3$ - Quadratic Equation

$= \left(x^2 - x\right) - 3$ - Group First 2 terms

$= \left[(x)^2 + 2x\left(-\frac{1}{2}\right)\right] - 3$ - Find Blue

$= \left[\underbrace{(x)^2 + 2x\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2}_{\downarrow} \underbrace{- \left(-\frac{1}{2}\right)^2}_{\downarrow}\right] - 3$ - Complete the Square by adding and subtracting $\left(-\frac{1}{2}\right)^2$

$= \left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}\right] - 3$ - Make the Perfect Square $(x)^2 + 2x\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 = \left(x - \frac{1}{2}\right)^2$

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 3$ - Remove "[ ]" Brackets

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{3}{1} \times \frac{4}{4}$ - Make a Common Denominator

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{12}{4}$ - Simplify

$= \left(x - \frac{1}{2}\right)^2 - \frac{13}{4}$ - Now you have Vertex Form

$\therefore$ The Vertex is $\left(+\frac{1}{2}, -\frac{13}{4}\right)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Complete the Square $y=a(x-h)^2+k$ the Vertex is (h,k)

$y=2x^2+8x+9$ - Quadratic Equation

$=2(x^2+4x)+9$ - Group First 2 terms

$=2\left[(x)^2+2x(+2)\right]+9$ - Find Blue

$=2\left[\underbrace{(x)^2+2x(+2)+(+2)^2}_{\downarrow}\underbrace{-(+2)^2}_{\downarrow}\right]+9$ - Complete the Square by adding and subtracting $(+2)^2$

$=2\left[\quad(x+2)^2\quad-\quad4\quad\right]+9$ - Make the Perfect Square $(x)^2+2x(+2)+(+2)^2=(x+2)^2$

$=2(x+2)^2-4(2)+9$ - Remove "[ ]" Brackets

$=2(x+2)^2-8+9$ - Simplify

$=2(x+2)^2+1$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-2,1)$

Example 1 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y=2x^2+8x+9$ - Quadratic Equation

$=2(x^2+4x)+9$ - Group First 2 terms

$=2\left[(x)^2+2x(+2)\right]+9$ - Find Blue

$=2\left[\underbrace{(x)^2+2x(+2)+(+2)^2}_{\downarrow}\underbrace{-(+2)^2}_{\downarrow}\right]+9$ - Complete the Square by adding and subtracting $(+2)^2$

$=2\left[\quad(x+2)^2\quad-\quad4\quad\right]+9$ - Make the Perfect Square $(x)^2+2x(+2)+(+2)^2=(x+2)^2$

$=2(x+2)^2-4(2)+9$ - Remove "[ ]" Brackets

$=2(x+2)^2-8+9$ - Simplify

$=2(x+2)^2+1$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-2,1)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 2 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = 4x^2 + 24x + 7$ - Quadratic Equation

$= 4\left(x^2 + 6x\right) + 7$ - Group First 2 terms

$= 4\left[(x)^2 + 2x(+3)\right] + 7$ - Find Blue

$= 4\left[\underbrace{(x)^2 + 2x(+3) + (+3)^2}_{\downarrow} \underbrace{-(+3)^2}_{\downarrow}\right] + 7$ - Complete the Square by adding and subtracting $(+3)^2$

$= 4\left[\quad (x+3)^2 \quad - \quad 9 \quad\right] + 7$ - Make the Perfect Square $(x)^2 + 2x(+3) + (+3)^2 = (x+3)^2$

$= 4(x+3)^2 - 9(4) + 7$ - Remove "[ ]" Brackets

$= 4(x+3)^2 - 36 + 7$ - Simplify

$= 4(x+3)^2 - 29$ - Now you have Vertex Form

$\therefore$ The Vertex is $(-3, -29)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 3 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = -3x^2 + 36x + 7$ - Quadratic Equation

$= -3\left(x^2 - 12x\right) + 7$ - Group First 2 terms

$= -3\left[(x)^2 + 2x(-6)\right] + 7$ - Find Blue

$= -3\left[\underbrace{(x)^2 + 2x(-6) + (-6)^2}_{\downarrow} \underbrace{-(-6)^2}_{\downarrow}\right] + 7$ - Complete the Square by adding and subtracting $(-6)^2$

$= -3\left[\quad (x-6)^2 \quad - \quad 36 \quad\right] + 7$ - Make the Perfect Square $(x)^2 + 2x(-6) + (-6)^2 = (x-6)^2$

$= -3(x-6)^2 - 36(-3) + 7$ - Remove "[ ]" Brackets

$= -3(x-6)^2 + 108 + 7$ - Simplify

$= -3(x-6)^2 + 115$ - Now you have Vertex Form

$\therefore$ The Vertex is $(+6, 115)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 4 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = -6x^2 + 12x + 5$ - Quadratic Equation

$= -6\left(x^2 - 2x\right) + 5$ - Group First 2 terms

$= -6\left[(x)^2 + 2x(-1)\right] + 5$ - Find Blue

$= -6\left[\underbrace{(x)^2 + 2x(-1) + (-1)^2}_{\downarrow} \underbrace{- (-1)^2}_{\downarrow}\right] + 5$ - Complete the Square by adding and subtracting $(-1)^2$

$= -6\left[\quad (x-1)^2 \quad - \quad 1 \quad\right] + 5$ - Make the Perfect Square $(x)^2 + 2x(-1) + (-1)^2 = (x-1)^2$

$= -6(x-1)^2 - 1(-6) + 5$ - Remove "[ ]" Brackets

$= -6(x-1)^2 + 6 + 5$ - Simplify

$= -6(x-1)^2 + 11$ - Now you have Vertex Form

$\therefore$ The Vertex is $(+1, 11)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 5 - The red template is the **same for EVERY QUESTION!!!**
- Blue is the same everywhere in the question.

$y = 2x^2 + 10x - 3$ - Quadratic Equation

$= 2\left(x^2 + 5x\right) - 3$ - Group First 2 terms

$= 2\left[(x)^2 + 2x\left(+\frac{5}{2}\right)\right] - 3$ - Find Blue

$= 2\left[\underbrace{(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2}_{\downarrow} \underbrace{- \left(+\frac{5}{2}\right)^2}_{\downarrow}\right] - 3$ - Complete the Square by adding and subtracting $\left(+\frac{5}{2}\right)^2$

$= 2\left[\quad \left(x + \frac{5}{2}\right)^2 \quad - \quad \frac{25}{4} \quad\right] - 3$ - Make the Perfect Square $(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2$

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}(2) - 3$ - Remove "[ ]" Brackets

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{25}{2} - \frac{3}{1} \times \frac{2}{2}$ - Make a Common Denominator

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{25}{2} - \frac{6}{2}$ - Simplify

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{31}{2}$ - Now you have Vertex Form

$\therefore$ The Vertex is $\left(-\frac{5}{2}, -\frac{31}{2}\right)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.

Example 6 - The red template is the **same for EVERY QUESTION!!!**

- Blue is the same everywhere in the question.

$y = -3x^2 - 15x - 3$ - Quadratic Equation

$= -3\left(x^2 + 5x\right) - 3$ - Group First 2 terms

$= -3\left[(x)^2 + 2x\left(+\frac{5}{2}\right)\right] - 3$ - Find Blue

$= -3\left[\underbrace{(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2}_{\downarrow} \underbrace{- \left(+\frac{5}{2}\right)^2}_{\downarrow}\right] - 3$ - Complete the Square by adding and subtracting $\left(+\frac{5}{2}\right)^2$

$= -3\left[\left(x+\frac{5}{2}\right)^2 - \frac{25}{4}\right] - 3$ - Make the Perfect Square $(x)^2 + 2x\left(+\frac{5}{2}\right) + \left(+\frac{5}{2}\right)^2 = \left(x+\frac{5}{2}\right)^2$

$= -3\left(x+\frac{5}{2}\right)^2 - \frac{25}{4}(-3) - 3$ - Remove "[ ]" Brackets

$= -3\left(x+\frac{5}{2}\right)^2 + \frac{75}{4} - \frac{3}{1}\times\frac{4}{4}$ - Make a Common Denominator

$= -3\left(x+\frac{5}{2}\right)^2 + \frac{75}{4} - \frac{12}{2}$ - Simplify

$= -3\left(x+\frac{5}{2}\right)^2 - \frac{63}{4}$ - Now you have Vertex Form

$\therefore$ The Vertex is $\left(-\frac{5}{2}, -\frac{63}{4}\right)$

- The Blue numbers are the same everywhere in a question.
- Completing the square could be called "Just find Blue". As soon as you know the value of Blue then you have the answer.