A1) Find the value of $\lim_{x\to 2}\dfrac{x^2-2x}{x^3+x^2-6x}$. [Chapter 2]

A3) If $f(x)=\dfrac{4x-1}{2x+3}$, find the expression for $f^{-1}(x)$. [Chapter 1]

A4) Find the value of $\lim_{x\to -\infty}\dfrac{\sqrt{7x^6-2x}}{3x^3-9}$. [Chapter 2]

A5) What is the equation for the horizontal asymptote of the curve $y=\dfrac{5x^3-2\sqrt{x}+1}{3+x^2-4x^3}$? [Chapter 2]

A6) If $f(x)=\sin x\cos x$, find the value of $f''\left(\dfrac{\pi}{4}\right)$. [Chapter 3]

A9) Find the value of $\lim_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$. [Chapter 2]

A10) If $y=\tan^{-1}\left(\sqrt{1+x^2}-x\right)$, find the expression of $\dfrac{dy}{dx}$. [Chapter 3]

B1) Find $f'(x)$ from first principles (i.e. by using only the definition of the derivative) for the following functions: [Chapter 2]

a) $f(x)=\dfrac{1}{x}$

b) $f(x)=\sin x$

B2) Find any suitable method to find $\dfrac{dy}{dx}$ for each of the following: [Chapter 3]

a) $y=\left(2+x^2\right)\tan x$

b) $y=\dfrac{e^{3x}}{2+x^3}$

c) $y=\sec\left(1+\sqrt{x}\right)$

d) $y=3^{\sin^{-1}2x}$

B3) Find the line which passes through the point $(0,-32)$ and tangent to the curve $y=2x^3$ at some point. [Chapter 2]

B4) Find $\dfrac{dy}{dx}$ if $y^3+2y^2+xy-2y+x=1$. What is the value of $\dfrac{dy}{dx}$ when $x=3$? [Chapter 3]

B5)

a) Let $f(x)=\begin{cases} x+1 & \text{if } x\ge 0 \\ x+2 & \text{if } x<0 \end{cases}$. Does $f'(0)$ exist? Justify your answer. [Chapter 2]

b) Let $f\left(x\right)=\begin{cases}\dfrac{c\left(x^2-x-2\right)}{x-2} & \text{if } x>2\\ cx^2+1 & \text{if } x\le 2\end{cases}$. Find a value of *c* such that *f* is continuous at 2. If such a value does not exist, explain why. [Chapter 2]

B6) Find the value of $\lim\limits_{x\to 1}\dfrac{x^{\frac{1}{7}}-1}{x^{\frac{1}{5}}-1}$. [Chapter 3]

A1) Find the value of $\lim_{x\to 2}\frac{x^2-2x}{x^3+x^2-6x}$. [Chapter 2]

$$\lim_{x\to 2}\frac{x^2-2x}{x^3+x^2-6x}$$

- Let's factorize the numerator and the denominator and see what cancels out.

$$=\lim_{x\to 2}\frac{x(x-2)}{x(x^2+x-6)}$$

$$=\lim_{x\to 2}\frac{x(x-2)}{x(x+3)(x-2)}$$

- Cancel out the common factors.

$$=\lim_{x\to 2}\frac{1}{x+3}$$

- Now we can substitute in $x=2$ to evaluate the value of the limit.

$$=\frac{1}{2+3}$$

$$=\underline{\underline{\frac{1}{5}}}$$

A3) If $f\left(x\right)=\frac{4x-1}{2x+3}$, find the expression for $f^{-1}\left(x\right)$. [Chapter 1]

$x=\frac{4y-1}{2y+3}$ - To find the inverse function, we swap $f\left(x\right)$ for *x* and *x* for *y*, and then solve for *y*.

$$x\left(2y+3\right)=4y-1$$
$$2xy+3x=4y-1$$
$$4y-2xy=1+3x$$
$$\left(4-2x\right)y=1+3x$$
$$y=\underline{\underline{\frac{1+3x}{4-2x}}}$$

A4) Find the value of $\lim\limits_{x\to-\infty}\dfrac{\sqrt{7x^6-2x}}{3x^3-9}$. [Chapter 2]

$$\lim_{x\to-\infty}\frac{\sqrt{7x^6-2x}}{3x^3-9}$$

- Both the numerator and denominator are polynomials, so we divide top and bottom by the largest power of *x*, in this case x^3.

$$=\lim_{x\to-\infty}\left(\frac{\sqrt{7x^6-2x}}{3x^3-9}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

- Note that since $x\to-\infty$, it is a negative number, so that we need to make sure we preserve the negative sign on the numerator when multiplying into the square root, which gives only positive values.

$$=\lim_{x\to-\infty}\frac{-\sqrt{\frac{1}{x^6}\left(7x^6-2x\right)}}{3-\frac{9}{x^3}}$$

$$=\lim_{x\to-\infty}\frac{-\sqrt{7-\frac{2}{x^6}}}{3-\frac{9}{x^3}}$$

- Now we can substitute in $x=-\infty$ to evaluate the value of the limit.

$$=\frac{-\sqrt{7-\frac{2}{(-\infty)^6}}}{3-\frac{9}{(-\infty)^3}}$$

- Any number divided by infinity is 0.

$$=\frac{-\sqrt{7-0}}{3-0}$$

$$=-\frac{\sqrt{7}}{3}$$

A5) What is the equation for the horizontal asymptote of the curve $y = \dfrac{5x^3 - 2\sqrt{x} + 1}{3 + x^2 - 4x^3}$? [Chapter 2]

$$\lim_{x\to\infty} \frac{5x^3 - 2\sqrt{x} + 1}{3 + x^2 - 4x^3}$$

- A horizontal asymptote exists if a function approaches a certain value asymptotically, i.e., the function approaches a certain value as $x \to \infty$. Both the numerator and denominator are polynomials, so we divide top and bottom by the largest power of *x*, in this case x^3, in order to determine the value of this limit.

$$= \lim_{x\to\infty} \left(\frac{5x^3 - 2\sqrt{x} + 1}{3 + x^2 - 4x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \right)$$

$$= \lim_{x\to-\infty} \frac{5 - \frac{2}{x^{\frac{5}{2}}} + \frac{1}{x^3}}{\frac{3}{x^3} + \frac{1}{x} - 4}$$

- Now we can substitute in $x = \infty$ to evaluate the value of the limit.

$$= \frac{5 - \frac{2}{\infty^{\frac{5}{2}}} + \frac{1}{\infty^3}}{\frac{3}{\infty^3} + \frac{1}{\infty} - 4}$$

- Any number divided by infinity is 0.

$$= \frac{5 - 0 + 0}{0 + 0 - 4}$$

$$= -\frac{5}{4}$$

Therefore the horizontal asymptote is $\underline{\underline{y = -\frac{5}{4}}}$

A6) If $f(x) = \sin x \cos x$, find the value of $f''\left(\frac{\pi}{4}\right)$. [Chapter 3]

$$f'(x) = \frac{d}{dx}(\sin x \cos x)$$

- While the derivative can be taken directly using the Product Rule, it is simpler if we rewrite the expression using the trigonometric identity $\sin 2x = 2\sin x \cos x$.

$$= \frac{d}{dx}\left(\frac{1}{2}\sin 2x\right)$$

- The Chain Rule is needed for this derivative.

$$= \frac{1}{2}\cos 2x \frac{d}{dx}2x$$

$$= \cos 2x$$

$$f''(x) = \frac{d}{dx}\cos 2x$$

- The Chain Rule is again needed.

$$= -\sin 2x \frac{d}{dx}2x$$

$$= -2\sin 2x$$

$$f''\left(\frac{\pi}{4}\right) = -2\sin\left(2 \cdot \frac{\pi}{4}\right)$$

- Now we can substitute in $x = \frac{\pi}{4}$ to evaluate $f''\left(\frac{\pi}{4}\right)$.

$$= -2\sin\left(\frac{\pi}{2}\right)$$

$$= -2(1)$$

$$= \underline{\underline{-2}}$$

A9) Find the value of $\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$. [Chapter 2]

$$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$$

- Both the numerator and denominator evaluate to 0. Since the numerator is a subtraction, we can rewrite the expression by multiplying top and bottom by $\sqrt{1+\tan x}+\sqrt{1+\sin x}$ to get a difference of squares on top.

$$=\lim_{x\to 0}\left(\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}\cdot\frac{\sqrt{1+\tan x}+\sqrt{1+\sin x}}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right)$$

$$=\lim_{x\to 0}\frac{(1+\tan x)-(1-\sin x)}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\tan x-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

- One of the few limits we know that involves x and trigonometric functions is $\lim\limits_{x\to 0}(\sin x/x)=1$. Let's therefore aim to get a $\sin^3 x$ term in the numerator to pair with the x^3 in the denominator by first writing $\tan x=\sin x/\cos x$. Note that the rest of the denominator evaluates to a defined value when applying the limit, so we write it as a separate factor that requires no further attention.

$$=\lim_{x\to 0}\left(\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right)$$

- We can pull out $1/\cos x$ from the numerator as a common factor, and see if the remainder can be written as $\sin^3 x$.

$$=\lim_{x\to 0}\left(\frac{1}{\cos x}\cdot\frac{\sin x-\sin x\cos x}{x^3}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right)$$

- $\sin x$ is a common factor, so we pull that out as well.

$$=\lim_{x\to 0}\left[\frac{1}{\cos x}\cdot\frac{\sin x(1-\cos x)}{x^3}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$

- We now want $1-\cos x$ to become $\sin^2 x$, which we can do by recalling that $\sin^2 x=1-\cos^2 x$, which is a difference of squares. We can therefore get what we want if we multiply top and bottom by $1+\cos x$.

$$=\lim_{x\to 0}\left[\frac{1}{\cos x}\cdot\frac{\sin x(1-\cos x)}{x^3}\cdot\frac{1+\cos x}{1+\cos x}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$

$$=\lim_{x\to 0}\left[\frac{1}{\cos x}\cdot\frac{\sin x(1-\cos^2 x)}{x^3}\cdot\frac{1}{1+\cos x}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$

$$= \lim_{x\to 0}\left[\frac{1}{\cos x}\cdot\frac{\sin x\left(\sin^2 x\right)}{x^3}\cdot\frac{1}{1+\cos x}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$

$$= \lim_{x\to 0}\left[\frac{1}{\cos x}\cdot\left(\frac{\sin x}{x}\right)^3\cdot\frac{1}{1+\cos x}\cdot\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$

- We can now distribute the limit to get 4 terms.

$$= \lim_{x\to 0}\frac{1}{\cos x}\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^3\cdot\lim_{x\to 0}\frac{1}{1+\cos x}\cdot\lim_{x\to 0}\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}$$

- Evaluate the value for all the terms.

$$= \frac{1}{\cos(0)}\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^3\cdot\frac{1}{1+\cos(0)}\cdot\frac{1}{\sqrt{1+\tan(0)}+\sqrt{1+\sin(0)}}$$

$$= \frac{1}{1}\cdot(1)^3\cdot\frac{1}{1+1}\cdot\frac{1}{\sqrt{1+0}+\sqrt{1+0}}$$

$$= 1\cdot 1\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \underline{\underline{\frac{1}{4}}}$$

A10) If $y = \tan^{-1}\left(\sqrt{1+x^2} - x\right)$, find the expression of $\dfrac{dy}{dx}$. [Chapter 3]

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}\left(\sqrt{1+x^2} - x\right)\right]$$

- We can take this derivative by using the Chain Rule. Recall that $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$.

$$= \frac{1}{1+\left(\sqrt{1+x^2} - x\right)^2} \cdot \frac{d}{dx}\left(\sqrt{1+x^2} - x\right)$$

- When taking the derivative of the square root term, the Chain Rule is needed once more.

$$= \frac{1}{1+\left(\sqrt{1+x^2} - x\right)^2} \cdot \left(\underbrace{\frac{1}{2\sqrt{1+x^2}}\frac{d}{dx}x^2}_{\text{Chain Rule}} - 1\right)$$

$$= \frac{1}{1+\left(\sqrt{1+x^2} - x\right)^2} \cdot \left(\frac{2x}{2\sqrt{1+x^2}} - 1\right)$$

- Now we need to simplify this expression. Let's expand out the squared term in the 1st fraction, while writing the subtraction inside the brackets as a single fraction.

$$= \frac{1}{1+\left(\sqrt{1+x^2}\right)^2 - 2x\sqrt{1+x^2} + x^2} \cdot \frac{2x - 2\sqrt{1+x^2}}{2\sqrt{1+x^2}}$$

$$= \frac{1}{1+\left(1+x^2\right) - 2x\sqrt{1+x^2} + x^2} \cdot \frac{2x - 2\sqrt{1+x^2}}{2\sqrt{1+x^2}}$$

- We next remove the square root term in the denominator of the 2nd fraction by multiplying top and bottom by $\sqrt{1+x^2}$.

$$= \frac{1}{2+2x^2 - 2x\sqrt{1+x^2}} \cdot \frac{2x - 2\sqrt{1+x^2}}{2\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$= \frac{1}{2\left(1+x^2\right) - 2x\sqrt{1+x^2}} \cdot \frac{2x\sqrt{1+x^2} - 2\left(1+x^2\right)}{2\left(1+x^2\right)}$$

- Note that the numerator of the 2nd fraction is just the negative of the denominator of the 1st fraction.

$$= \frac{1}{2\left(1+x^2\right) - 2x\sqrt{1+x^2}} \cdot \frac{-\left[2x\sqrt{1+x^2} + 2\left(1+x^2\right)\right]}{2\left(1+x^2\right)}$$

- The numerator of the 2nd fraction therefore cancels out the denominator of the 1st fraction, leaving behind -1.

$$= \underline{\underline{\frac{1}{2\left(1+x^2\right)}}}$$

B1) Find $f'(x)$ from first principles (i.e. by using only the definition of the derivative) for the following functions: [Chapter 2]

a) $f(x)=\frac{1}{x}$

b) $f(x)=\sin x$

a)

$$f'(x)=\lim_{h\to x}\frac{f(h)-f(x)}{h-x}$$

- This is one way to write the definition of the derivative. Let's substitute in the function.

$$=\lim_{h\to x}\frac{\frac{1}{h}-\frac{1}{x}}{h-x}$$

- We can simplify this by writing the numerator as a single fraction.

$$=\lim_{h\to x}\left(\frac{h-x}{hx}\cdot\frac{1}{h-x}\right)$$

- Cancel out the common factor.

$$=\lim_{h\to x}\frac{1}{hx}$$

- We can now apply the limit.

$$=\frac{1}{xx}$$

$$=\underline{\underline{\frac{1}{x^2}}}$$

b)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

- This is one way to write the definition of the derivative. Let's substitute in the function.

$$=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

- We can expand this using the trigonometric identity $\sin(a+b)=\sin a\cos b+\cos a\sin b$.

$$=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

- We can then split this into 2 fractions.

$$=\lim_{h\to 0}\left(\frac{\sin x\cos h-\sin x}{h}+\frac{\cos x\sin h}{h}\right)$$

- These can be factorized to yield limits we know.

$$=\lim_{h\to 0}\left(\sin x\frac{\cos h-1}{h}+\cos x\frac{\sin h}{h}\right)$$

- Distribute the limit.

$$=\lim_{h\to 0}\sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\lim_{h\to 0}\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}$$

- Recall that $\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}=0$ and $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$.

$$=\sin x\cdot 0+\cos x\cdot 1$$

$$=\underline{\underline{\cos x}}$$

B2) Find any suitable method to find $\frac{dy}{dx}$ for each of the following: [Chapter 3]

a) $y=\left(2+x^2\right)\tan x$

b) $y=\frac{e^{3x}}{2+x^3}$

c) $y=\sec\left(1+\sqrt{x}\right)$

d) $y=3^{\sin^{-1}2x}$

a)

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(2+x^2\right)\tan x\right]$$

- The Product Rule is needed here.

$$=\left[\frac{d}{dx}\left(2+x^2\right)\right]\tan x+\left(2+x^2\right)\frac{d}{dx}\tan x$$

$$=\underline{\underline{2x\tan x+\left(2+x^2\right)\sec^2 x}}$$

b)

$$\frac{dy}{dx}=\frac{d}{dx}\frac{e^{3x}}{2+x^3}$$

- The Quotient Rule is needed here.

$$=\frac{\left(\frac{d}{dx}e^{3x}\right)\left(2+x^3\right)-e^{3x}\frac{d}{dx}\left(2+x^3\right)}{\left(2+x^3\right)^2}$$

- The Chain Rule is needed for the derivative on the left.

$$=\frac{\left(e^{3x}\frac{d}{dx}3x\right)\left(2+x^3\right)-3x^2e^{3x}}{\left(2+x^3\right)^2}$$

$$=\underline{\underline{\frac{3e^{3x}\left(2+x^3\right)-3x^2e^{3x}}{\left(2+x^3\right)^2}}}$$

c)

$$\frac{dy}{dx}=\frac{d}{dx}\sec\left(1+\sqrt{x}\right)$$

- The Chain Rule is needed here.

$$=\sec\left(1+\sqrt{x}\right)\tan\left(1+\sqrt{x}\right)\frac{d}{dx}\left(1+\sqrt{x}\right)$$

- Recall that $\frac{d}{dx}\sec x=\sec x\tan x$.

$$=\underline{\underline{\frac{1}{2\sqrt{x}}\sec\left(1+\sqrt{x}\right)\tan\left(1+\sqrt{x}\right)}}$$

d)

$$\frac{dy}{dx} = \frac{d}{dx} 3^{\sin^{-1} 2x}$$

- The Chain Rule is needed here. Recall that $\frac{d}{dx} a^x = a^x \ln a$.

$$= \left(3^{\sin^{-1} 2x} \ln 3\right) \frac{d}{dx} \sin^{-1} 2x$$

The Chain Rule is needed again here. Recall that

- $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$.

$$= \left(3^{\sin^{-1} 2x} \ln 3\right) \cdot \frac{1}{\sqrt{1-(2x)^2}} \frac{d}{dx} 2x$$

$$= \left(3^{\sin^{-1} 2x} \ln 3\right) \cdot \frac{2}{\sqrt{1-4x^2}}$$

B3) Find the line which passes through the point $(0,-32)$ and tangent to the curve $y=2x^3$ at some point. [Chapter 2]

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}2x^3\\&=6x^2\end{aligned}$$ - Let's first evaluate the derivative.

Let the point on the curve that the line is tangent to be $\left(a,2a^3\right)$.

$$\left.\frac{dy}{dx}\right|_{x=a}=\frac{2a^3-(-32)}{a-0}$$ - The slope of the tangent must be the same as the slope as calculated from the point on the curve and the outside point $(0,-32)$.

$$6a^2=\frac{2a^3+32}{a}$$ - Simplify and solve for *a*.

$$\begin{aligned}6a^3&=2a^3+32\\4a^3&=32\\a^3&=8\\a&=2\end{aligned}$$

$$y=\left.\frac{dy}{dx}\right|_{x=2}x+c$$ - Now we can write out the equation of the line.

$$=6\left(2^2\right)x+c$$ - To find the constant *c*, we can substitute in the point $(0,-32)$, which is on this line.

$$\begin{aligned}-32&=24(0)+c\\c&=-32\end{aligned}$$

Therefore the line is $\underline{\underline{y=24x-32}}$

B4) Find $\frac{dy}{dx}$ if $y^3+2y^2+xy-2y+x=1$. What is the value of $\frac{dy}{dx}$ when $x=3$? [Chapter 3]

$$y^3+2y^2+xy-2y+x=1$$

- Differentiate the equation implicitly.

$$\frac{d}{dx}\left(y^3+2y^2+xy-2y+x\right)=\frac{d}{dx}0$$

- Chain Rule and Product Rule are both needed.

$$\underbrace{3y^2\frac{dy}{dx}}_{\text{Chain Rule}}+\underbrace{4y\frac{dy}{dx}}_{\text{Chain Rule}}+\underbrace{\frac{dx}{dx}y+x\frac{dy}{dx}}_{\text{Product Rule}}-2\frac{dy}{dx}+1=0$$

$$3y^2\frac{dy}{dx}+4y\frac{dy}{dx}+y+x\frac{dy}{dx}-2\frac{dy}{dx}+1=0$$

- Solve for $\frac{dy}{dx}$.

$$\left(3y^2+4y+x-2\right)\frac{dy}{dx}=-1-y$$

$$\frac{dy}{dx}=\frac{-1-y}{3y^2+4y+x-2}$$

- We now need to determine value of y that corresponds to $x=3$.

$$y^3+2y^2+3y-2y+3=1$$

- Substitute $x=3$ into the original equation.

$$y^3+2y^2+y+2=0$$

- $y=-2$ yields 0 on the left side, so $y+2$ is a factor.

$$\left(y+2\right)\left(y^2+1\right)=0$$

- The quadratic factor gives imaginary roots, which can be ignored.

$$y=-2$$

$$\left.\frac{dy}{dx}\right|_{x=3,y=-2}=\frac{-1-\left(-2\right)}{3\left(-2\right)^2+4\left(-2\right)+3-2}$$

- We can now evaluate the derivative at $x=3$, $y=2$.

$$=\frac{1}{12-8+3-2}$$

$$=\underline{\underline{\frac{1}{5}}}$$

B5)

a) Let $f\left(x\right)=\begin{cases} x+1 & \text{if } x\geq 0 \\ x+2 & \text{if } x<0 \end{cases}$. Does $f'\left(0\right)$ exist? Justify your answer. [Chapter 2]

b) Let $f\left(x\right)=\begin{cases} \dfrac{c\left(x^2-x-2\right)}{x-2} & \text{if } x>2 \\ cx^2+1 & \text{if } x\leq 2 \end{cases}$. Find a value of *c* such that *f* is continuous at 2. If such a value does not exist, explain why. [Chapter 2]

a)

$$\begin{aligned} \lim_{x\to 0^+} f\left(x\right) &= \lim_{x\to 0^+}\left(x+1\right) \\ &= 0+1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \lim_{x\to 0^-} f\left(x\right) &= \lim_{x\to 0^+}\left(x+2\right) \\ &= 0+2 \\ &= 2 \end{aligned}$$

Since $\lim_{x\to 0^+} f\left(x\right)\neq \lim_{x\to 0^-} f\left(x\right)$, $f\left(x\right)$ is not continuous at $x=0$, therefore $f'\left(0\right)$ **does not exist**.

b)

$$\lim_{x\to 2^+}\frac{c\left(x^2-x-2\right)}{x-2}=\lim_{x\to 2^-}\left(cx^2+1\right)$$

- We need to make sure the function is continuous when $x=2$. We can then simplify the left side by factorizing the numerator.

$$\lim_{x\to 2^+}\frac{c\left(x-2\right)\left(x+1\right)}{x-2}=\lim_{x\to 2^-}\left(cx^2+1\right)$$

$$\lim_{x\to 2^+} c\left(x+1\right)=\lim_{x\to 2^-}\left(cx^2+1\right)$$

- We now substitute $x=2$ into the expression.

$$c\left(2+1\right)=c\left(2^2\right)+1$$

- Solve for *c*.

$$\begin{aligned} 3c &= 4c+1 \\ c &= \underline{\underline{-1}} \end{aligned}$$

B6) Find the value of $\lim_{x\to 1}\frac{x^{\frac{1}{7}}-1}{x^{\frac{1}{5}}-1}$ **without** using l'Hopital's Rule. [Chapter 3]

$$\lim_{x\to 1}\frac{x^{\frac{1}{7}}-1}{x^{\frac{1}{5}}-1}$$

- Since this is a fraction, we shall rewrite the expression so to match the definition of the derivative. Recall that the definition of the derivative can be written as $f'(x)=\lim_{x\to a}\left[f(x)-f(a)\right]/(x-a)$. This means that we need to have $x-1$ in the denominator since the limit is for $x\to 1$. There is, however, no simple multiplicative factor that can do this. Instead, we need to make a substitution that replaces $x^{1/5}$. If we let $u=x^{1/5}$, then $x^{1/7}=\left(x^{1/5}\right)^{7/5}=u^{7/5}$, and the limit $x\to 1$ becomes $u\to 1^{7/5}=1$.

$$=\lim_{u\to 1}\frac{u^{\frac{5}{7}}-1}{u-1}$$

- Now that we have the denominator matching that of the definition of the derivative, we need to match the numerator as well. Note that $1^{7/5}=1$.

$$=\lim_{u\to 1}\frac{u^{\frac{5}{7}}-1^{\frac{5}{7}}}{u-1}$$

- We can now see that our expression matches the definition of the derivative, with $f(u)=u^{5/7}$, while $a=1$.

$$=\lim_{u\to a}\frac{f(u)-f(a)}{u-a}$$

$$=f'(a)$$

- Now we can evaluate the derivative.

$$=\left.\frac{d}{du}u^{\frac{5}{7}}\right|_{u=1}$$

$$=\left.\frac{5}{7}u^{-\frac{2}{7}}\right|_{u=1}$$

- Finally, we substitute in $u=1$.

$$=\frac{5}{7}\left(1^{-\frac{2}{7}}\right)$$

$$=\underline{\underline{\frac{5}{7}}}$$