

A3) Find the value of  $\lim_{x \rightarrow 0} \frac{\sin(2x)\cos(3x)\sin(4x)}{x\cos(5x)\sin(6x)}$ . [Chapter 3]

A8) Find the value of  $\lim_{x \rightarrow 0} (\cot x - \csc x)$ . [Chapter 3]

A9) If  $x^2 + xy + y^3 = 1$ , find the value of  $\frac{d^3y}{dx^3}$  at the point where  $x = 1$ . [Chapter 3]

A10) There exists one, and only one, straight line that is tangent to the curve  $y = x^4 - 9x^2$  at two points. Find the distance between these two points. [Chapter 2]

B1) Find  $f'(x)$  from first principles (i.e. by using only the definition of the derivative) for  $f(x) = x^3$ . [Chapter 2]

B2) Find any suitable method to find  $\frac{dy}{dx}$  for each of the following: [Chapter 3]

a)  $y = (4 + x^2)\sin^{-1} x$ ,  $-1 < x < 1$

b)  $y = \frac{\sqrt{x}}{5 + e^x}$

c)  $y = \tan^{-1} x^2$

d)  $y = 2^{\sin 5x}$

B3) An object is thrown upward so its height above ground after  $t$  seconds is  $50 + 96t - 16t^2$  feet. Find the velocity of the object when it is 178 feet above ground on its way up. [Chapter 3]

B4) Find the line passing through the point  $(-3, 0)$  and tangent to the curve  $y = \sqrt{x-1}$  at some point. [Chapter 2]

B5) Let  $f(x) = \begin{cases} 2x + c & \text{if } x < 3 \\ cx + c + 1 & \text{if } x \geq 3 \end{cases}$ . Find the value of  $c$  so that  $f$  is continuous everywhere. [Chapter 2]

B6) Find the values of the constants  $a$  and  $b$  such that  $\lim_{x \rightarrow 0^-} \frac{\sqrt{-a|x| + b} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$ . [Chapter 2]

A3) Find the value of  $\lim_{x \rightarrow 0} \frac{\sin(2x)\cos(3x)\sin(4x)}{x\cos(5x)\sin(6x)}$ . [Chapter 3]

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin(2x)\cos(3x)\sin(4x)}{x\cos(5x)\sin(6x)} \\ &= \lim_{x \rightarrow 0} \left[ \frac{\sin(2x)}{x} \cdot \frac{\cos(3x)}{\cos(5x)} \cdot \frac{\sin(4x)}{\sin(6x)} \right] \\ &= \lim_{x \rightarrow 0} \frac{\sin(2x)}{x} \cdot \lim_{x \rightarrow 0} \frac{\cos(3x)}{\cos(5x)} \cdot \lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(6x)} \\ &= \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sin(2x)}{\frac{d}{dx} x} \cdot \lim_{x \rightarrow 0} \frac{\cos(3x)}{\cos(5x)} \cdot \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sin(4x)}{\frac{d}{dx} \sin(6x)} \\ &= \lim_{x \rightarrow 0} \frac{\cos(2x) \frac{d}{dx} 2x}{1} \cdot \lim_{x \rightarrow 0} \frac{\cos(3x)}{\cos(5x)} \cdot \lim_{x \rightarrow 0} \frac{\cos(4x) \frac{d}{dx} 4x}{\cos(6x) \frac{d}{dx} 6x} \\ &= \lim_{x \rightarrow 0} [2 \cos(2x)] \cdot \lim_{x \rightarrow 0} \frac{\cos(3x)}{\cos(5x)} \cdot \lim_{x \rightarrow 0} \frac{4 \cos(4x)}{6 \cos(6x)} \\ &= 2 \cos(2 \cdot 0) \cdot \frac{\cos(3 \cdot 0)}{\cos(5 \cdot 0)} \cdot \frac{4 \cos(4 \cdot 0)}{6 \cos(6 \cdot 0)} \\ &= 2 \cdot 1 \cdot \frac{1}{1} \cdot \frac{4 \cdot 1}{6 \cdot 1} \\ &= \underline{\underline{\frac{4}{3}}} \end{aligned}$$

- We first write this fraction into a product of convenient fractions.

- We can now distribute the limit to get 3 individual limits to evaluate.

- The 1<sup>st</sup> and 3<sup>rd</sup> limits are in the form 0/0, so we apply l'Hopital's Rule: take the derivative top and bottom separately.

- The Chain Rule is required for each of the derivative of sine.

- Now we can substitute in  $x = 0$  to evaluate the value of the limit.

A8) Find the value of  $\lim_{x \rightarrow 0} (\cot x - \csc x)$ . [Chapter 3]

$$\begin{aligned} & \lim_{x \rightarrow 0} (\cot x - \csc x) && \text{- We first write the expression in terms of sines and cosines.} \\ & = \lim_{x \rightarrow 0} \left( \frac{\cos x}{\sin x} - \frac{1}{\sin x} \right) && \text{- Write as a single fraction.} \\ & = \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x} && \text{- The limit is in the form } 0/0 \text{, so we apply l'Hopital's Rule: take the derivative top and bottom separately.} \\ & = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\cos x - 1)}{\frac{d}{dx} \sin x} \\ & = \lim_{x \rightarrow 0} \frac{-\sin x}{\cos x} && \text{- This is just the tangent.} \\ & = -\lim_{x \rightarrow 0} \tan x && \text{- Now we can substitute in } x = 0 \text{ to evaluate the value of the limit.} \\ & = 0 \end{aligned}$$

A9) If  $x^2 + xy + y^3 = 1$ , find the value of  $\frac{d^3y}{dx^3}$  at the point where  $x = 1$ . [Chapter 3]

$$x^2 + xy + y^3 = 1 \quad - \text{ We take the derivative on both sides}$$

$$\frac{d}{dx}(x^2 + xy + y^3) = \frac{d}{dx}1 \quad - \text{ The Product Rule is needed for the 2}^{\text{nd}} \text{ term on the left.}$$

$$2x + \underbrace{\frac{dx}{dx}y + x\frac{dy}{dx}}_{\text{Product Rule}} + 3y^2\frac{dy}{dx} = 0$$

$$2x + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0 \quad - \text{ Let's call this Equation 1. Solve for } \frac{dy}{dx}.$$

$$\begin{aligned} (x + 3y^2)\frac{dy}{dx} &= -2x - y \\ \frac{dy}{dx} &= \frac{-2x - y}{x + 3y^2} \end{aligned}$$

$$0 = \frac{d}{dx}\left(2x + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx}\right)$$

- We next take the derivative on both sides of Equation 1.

$$\frac{d}{dx}0 = \frac{d}{dx}\left(2x + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx}\right)$$

- The Product Rule is needed for the 3<sup>rd</sup> and 4<sup>th</sup> terms on the right.

$$0 = 2 + \frac{dy}{dx} + \underbrace{\frac{dx}{dx}\frac{dy}{dx} + x\frac{d^2y}{dx^2}}_{\text{Product Rule}} + \underbrace{\left[\frac{d}{dx}3y^2\right]\frac{dy}{dx} + 3y^2\frac{d^2y}{dx^2}}_{\text{Product Rule}}$$

- The Chain Rule is needed for term in square brackets.

$$0 = 2 + \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \underbrace{\left[6y\frac{dy}{dx}\right]\frac{dy}{dx}}_{\text{Chain Rule}} + 3y^2\frac{d^2y}{dx^2}$$

$$0 = 2 + 2\frac{dy}{dx} + x\frac{d^2y}{dx^2} + 6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2}$$

- Let's call this Equation 2. Solve for  $\frac{d^2y}{dx^2}$ .

$$(x + 3y^2)\frac{d^2y}{dx^2} = -2 - 2\frac{dy}{dx} - 6y\left(\frac{dy}{dx}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-2 - 2\frac{dy}{dx} - 6y\left(\frac{dy}{dx}\right)^2}{x + 3y^2}$$

$$0 = 2 + 2\frac{dy}{dx} + x\frac{d^2y}{dx^2} + 6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2}$$

- We next take the derivative on both sides of Equation 2.

$$\frac{d}{dx}0 = \frac{d}{dx}\left[2 + 2\frac{dy}{dx} + x\frac{d^2y}{dx^2} + 6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2}\right]$$

- The Product Rule is needed for the rightmost 3 terms.

$$0 = \left[ \begin{aligned} &2 \frac{d^2 y}{dx^2} + \underbrace{\frac{dx}{dx} \frac{d^2 y}{dx^2} + x \frac{d^3 y}{dx^3}}_{\text{Product Rule}} \\ &+ 6 \underbrace{\frac{dy}{dx} \left( \frac{dy}{dx} \right)^2}_{\text{Product Rule}} + 6y \underbrace{\frac{d}{dx} \left( \frac{dy}{dx} \right)^2}_{\text{Product Rule}} + \left( \frac{d}{dx} 3y^2 \right) \frac{d^2 y}{dx^2} + 3y^2 \frac{d^3 y}{dx^3} \end{aligned} \right]$$

- The Chain Rule is needed in derivatives involving y.

$$0 = \left[ \begin{aligned} &2 \frac{d^2 y}{dx^2} + \frac{d^2 y}{dx^2} + x \frac{d^3 y}{dx^3} + 6 \left( \frac{dy}{dx} \right)^3 \\ &+ 6y \underbrace{\left( 2 \frac{dy}{dx} \frac{d^2 y}{dx^2} \right)}_{\text{Chain Rule}} + \underbrace{\left( 6y \frac{dy}{dx} \right)}_{\text{Chain Rule}} \frac{d^2 y}{dx^2} + 3y^2 \frac{d^3 y}{dx^3} \end{aligned} \right]$$

$$0 = 3 \frac{d^2 y}{dx^2} + 6 \left( \frac{dy}{dx} \right)^3 + 18y \frac{dy}{dx} \frac{d^2 y}{dx^2} + (x + 3y^2) \frac{d^3 y}{dx^3}$$

- Let's call this Equation 3. Solve for  $d^3 y/dx^2$ .

$$(x + 3y^2) \frac{d^3 y}{dx^3} = -3 \frac{d^2 y}{dx^2} - 6 \left( \frac{dy}{dx} \right)^3 - 18y \frac{dy}{dx} \frac{d^2 y}{dx^2}$$

$$\frac{d^3 y}{dx^3} = \frac{-3 \frac{d^2 y}{dx^2} - 6 \left( \frac{dy}{dx} \right)^3 - 18y \frac{dy}{dx} \frac{d^2 y}{dx^2}}{x + 3y^2}$$

$x^2 + xy + y^3 = 1$  - We next determine the value of y corresponding to  $x = 1$ .

$$1^2 + (1)y + y^3 = 1$$

$$y + y^3 = 0$$

$$y(1 + y^2) = 0$$

$$y = 0$$

$\frac{dy}{dx} = \frac{-2x - y}{x + 3y^2}$  - We can now determine the value of the 1<sup>st</sup> derivative at  $x = 1$ .

$$= \frac{-2(1) - 0}{1 + 3(0)^2}$$

$$= -2$$

$$\frac{d^2 y}{dx^2} = \frac{-2 - 2 \frac{dy}{dx} - 6y \left( \frac{dy}{dx} \right)^2}{x + 3y^2}$$

- We can now determine the value of the 2<sup>nd</sup> derivative at  $x = 1$ .

$$= \frac{-2 - 2(-2) - 6(0)(-2)^2}{1 + 3(0)^2}$$

$$= 2$$

$$\begin{aligned}\frac{d^3 y}{dx^3} &= \frac{-3 \frac{d^2 y}{dx^2} - 6 \left( \frac{dy}{dx} \right)^3 - 18y \frac{dy}{dx} \frac{d^2 y}{dx^2}}{x + 3y^2} && \text{- Lastly, we can now determine the value of the 3<sup>rd</sup> derivative at } x = 1. \\ &= \frac{-3(2) - 6(-2)^3 - 18(0)(-2)(2)}{1 + 3(0)^2} \\ &= -6 + 48 \\ &= \underline{\underline{42}}\end{aligned}$$

A10) There exists one, and only one, straight line that is tangent to the curve  $y = x^4 - 9x^2$  at two points. Find the distance between these two points. [Chapter 2]

$$y = x^4 - 9x^2 \quad - \text{ We first find the expression for the slope of the curve.}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x^4 - 9x^2) \\ &= 4x^3 - 18x \end{aligned}$$

$4x^3 - 18x = 0$  - Note that the curve is even (symmetric around the  $y$ -axis). If there is only 1 straight line that is tangent to the curve at 2 points, this line must be horizontal. The slope of this line must therefore be zero. We can then divide out the common factor 2.

$$2x^3 - 9x = 0 \quad - \text{ Factorize.}$$

$$x(2x^2 - 9) = 0$$

Therefore:

$$2x^2 - 9 = 0$$

$$x = 0 \quad \text{or} \quad x^2 = \frac{9}{2}$$

$$x = \pm \frac{3}{\sqrt{2}}$$

$d = \frac{3}{\sqrt{2}} - \frac{-3}{\sqrt{2}}$  - By the same symmetry argument, the two points must be at  $x = \pm 3/\sqrt{2}$  and not  $x = 0$ . The distance is just the difference between the  $x$ -coordinates, since the line is horizontal.

$$= \frac{6}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= \underline{\underline{3\sqrt{2}}}$$

B1) Find  $f'(x)$  from first principles (i.e. by using only the definition of the derivative) for  $f(x) = x^3$ . [Chapter 2]

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow x} \frac{f(h) - f(x)}{h - x} && \text{- This is one way to write the definition of the derivative. Let's substitute in the function.} \\ &= \lim_{h \rightarrow x} \frac{h^3 - x^3}{h - x} && \text{- We can simplify this by factorizing the numerator.} \\ &= \lim_{h \rightarrow x} \frac{(h - x)(h^2 + xh + x^2)}{h - x} && \text{- Cancel out the common factor.} \\ &= \lim_{h \rightarrow x} (h^2 + xh + x^2) && \text{- We can now apply the limit.} \\ &= x^2 + xx + x^2 \\ &= \underline{\underline{3x^2}} \end{aligned}$$

B2) Find any suitable method to find  $\frac{dy}{dx}$  for each of the following: [Chapter 3]

a)  $y = (4 + x^2) \sin^{-1} x$ ,  $-1 < x < 1$

b)  $y = \frac{\sqrt{x}}{5 + e^x}$

c)  $y = \tan^{-1} x^2$

d)  $y = 2^{\sin 5x}$

a)

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [(4 + x^2) \sin^{-1} x] && \text{- The Product Rule is needed here.} \\ &= \left[ \frac{d}{dx} (4 + x^2) \right] \sin^{-1} x + (4 + x^2) \frac{d}{dx} \sin^{-1} x \\ &= \underline{\underline{2x \sin^{-1} x + (4 + x^2) \frac{1}{\sqrt{1-x^2}}}} \end{aligned}$$

b)

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \frac{\sqrt{x}}{5 + e^x} && \text{- The Quotient Rule is needed here.} \\ &= \frac{\left( \frac{d}{dx} \sqrt{x} \right) (5 + e^x) - \sqrt{x} \frac{d}{dx} (5 + e^x)}{(5 + e^x)^2} \\ &= \underline{\underline{\frac{1}{2\sqrt{x}} (5 + e^x) - \sqrt{x} e^x}} \end{aligned}$$

c)

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \tan^{-1} x^2 && \text{- The Chain Rule is needed here.} \\ &= \frac{1}{1 + (x^2)^2} \frac{d}{dx} x^2 \\ &= \underline{\underline{\frac{2x}{1 + x^4}}} \end{aligned}$$

d)

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} 2^{\sin 5x} && \text{- The Chain Rule is needed here.} \\ &= (2^{\sin 5x} \ln 2) \frac{d}{dx} \sin 5x && \text{- The Chain Rule is needed again here.} \\ &= (2^{\sin 5x} \ln 2) \cos 5x \frac{d}{dx} 5x \\ &= \underline{\underline{5(2^{\sin 5x} \ln 2) \cos 5x}} \end{aligned}$$

B3) An object is thrown upward so its height above ground after  $t$  seconds is  $50 + 96t - 16t^2$  feet. Find the velocity of the object when it is 178 feet above ground on its way up. [Chapter 3]

$$v = \frac{d}{dt}(50 + 96t - 16t^2) \quad \text{- Velocity is the derivative of displacement with respect to time.}$$
$$= 96 - 32t$$

$$50 + 96t - 16t^2 = 178 \quad \text{- We next find the time corresponding to the object at 178ft on its way up.}$$

$$16t^2 - 96t + 128 = 0 \quad \text{- Divide out the common factor 16.}$$

$$t^2 - 6t + 8 = 0 \quad \text{- Factorize}$$

$$(t - 2)(t - 4) = 0 \quad \text{- Since the object is thrown up, the earlier time corresponding to it going up.}$$

$$t = 2$$

$$v(2) = 96 - 32(2) \quad \text{- Substitute } t = 2 \text{ into the expression for velocity.}$$

$$= 96 - 64$$

$$= \underline{\underline{32 \text{ (feet per second)}}}$$

B4) Find the line passing through the point  $(-3, 0)$  and tangent to the curve  $y = \sqrt{x-1}$  at some point. [Chapter 2]

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sqrt{x-1} \quad \text{- Let's first evaluate the derivative.} \\ &= \frac{1}{2\sqrt{x-1}}\end{aligned}$$

Let the point on the curve that the line is tangent to be  $(a, \sqrt{a-1})$ .

$$\left. \frac{dy}{dx} \right|_{x=a} = \frac{\sqrt{a-1} - 0}{a - (-3)} \quad \text{- The slope of the tangent must be the same as the slope as calculated from the point on the curve and the outside point } (-3, 0).$$

$$\frac{1}{2\sqrt{a-1}} = \frac{\sqrt{a-1}}{a+3} \quad \text{- Cross-multiply and solve for } a.$$

$$a+3 = 2\sqrt{a-1}\sqrt{a-1}$$

$$= 2(a-1)$$

$$= 2a - 2$$

$$a = 5$$

$$y = \left. \frac{dy}{dx} \right|_{x=5} x + c \quad \text{- Now we can write out the equation of the line.}$$

$$= \frac{1}{2\sqrt{5-1}} x + c \quad \text{- To find the constant } c, \text{ we can substitute in the point } (-3, 0), \text{ which is on this line.}$$

$$0 = \frac{1}{2(2)}(-3) + c$$

$$c = \frac{3}{4}$$

Therefore the line is  $y = \frac{1}{4}x + \frac{3}{4}$

B5)

a) Let  $f(x) = \begin{cases} 2x + c^2 & \text{if } x < 3 \\ cx + c + 2 & \text{if } x \geq 3 \end{cases}$ . Find the value of  $c$  so that  $f$  is continuous everywhere. [Chapter 2]

a)

$\lim_{x \rightarrow 3^-} (2x + c^2) = \lim_{x \rightarrow 3^+} (cx + c + 2)$  - Since both  $2x + c^2$  and  $cx + c + 2$  are continuous, the only place we need to make sure is continuous is when  $x = 3$ .

$2(3) + c^2 = 3c + c + 2$  - Move everything to one side and solve the quadratic.

$$c^2 - 4c + 4 = 0$$

$$(c - 2)^2 = 0$$

$$c = \underline{\underline{2}}$$

B6) Find the values of the constants  $a$  and  $b$  such that  $\lim_{x \rightarrow 0^-} \frac{\sqrt{-a|x|+b} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$ . [Chapter 2]

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{-a|x|+b} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$$

Let's first remove the absolute sign in the square root. Since the limit is towards zero from the negative,  $x$  is negative, thus  $|x| = -x$ .

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{-a(-x)+b} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{ax+b} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$$

Let's call this Equation 1. The denominator of this limit is 0. For the limit to be defined, it needs to be in the form  $0/0$ , i.e., the numerator has to be 0 as well.

$$\lim_{x \rightarrow 0^-} (\sqrt{ax+b} - \sqrt{3}) = 0$$

- Substitute in  $x = 0$  and solve for  $b$ .

$$\sqrt{a(0)+b} - \sqrt{3} = 0$$

$$\sqrt{b} = \sqrt{3}$$

$$b = \underline{\underline{3}}$$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{ax+3} - \sqrt{3}}{\sqrt{x+4} - 2} = 4$$

This is Equation 1. Let's multiply top and bottom by  $\sqrt{x+4} + 2$  to get a difference of squares at the bottom.

$$\lim_{x \rightarrow 0^-} \left( \frac{\sqrt{ax+3} - \sqrt{3}}{\sqrt{x+4} - 2} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \right) = 4$$

$$\lim_{x \rightarrow 0^-} \frac{(\sqrt{ax+3} - \sqrt{3})(\sqrt{x+4} + 2)}{(\sqrt{x+4})^2 - 2^2} = 4$$

$$\lim_{x \rightarrow 0^-} \frac{(\sqrt{ax+3} - \sqrt{3})(\sqrt{x+4} + 2)}{x+4-4} = 4$$

$$\lim_{x \rightarrow 0^-} \frac{(\sqrt{ax+3} - \sqrt{3})(\sqrt{x+4} + 2)}{x} = 4$$

This is still in the form  $0/0$ , so we multiply top and bottom by  $\sqrt{ax+3} + \sqrt{3}$  to make a non-zero denominator.

$$\lim_{x \rightarrow 0^-} \frac{(\sqrt{ax+3} - \sqrt{3})(\sqrt{x+4} + 2)}{x} \cdot \frac{\sqrt{ax+3} + \sqrt{3}}{\sqrt{ax+3} + \sqrt{3}} = 4$$

- It's another difference of squares on the numerator.

$$\lim_{x \rightarrow 0^-} \frac{\left[ (\sqrt{ax+3})^2 - (\sqrt{3})^2 \right] (\sqrt{x+4} + 2)}{x(\sqrt{ax+3} + \sqrt{3})} = 4$$

$$\lim_{x \rightarrow 0^-} \frac{(ax+3-3)(\sqrt{x+4} + 2)}{x(\sqrt{ax+3} + \sqrt{3})} = 4$$

$$\lim_{x \rightarrow 0^-} \frac{ax(\sqrt{x+4} + 2)}{x(\sqrt{ax+3} + \sqrt{3})} = 4$$

- Cancel out the common factor  $x$ .

$$\lim_{x \rightarrow 0^-} \frac{a\sqrt{x+4} + 2}{\sqrt{ax+3} + \sqrt{3}} = 4$$

- Apply the limit.

$$\frac{a(\sqrt{0+4}+2)}{\sqrt{a(0)+3}+\sqrt{3}} = 4 \quad - \text{Simplify and solve for } a.$$

$$\frac{4a}{2\sqrt{3}} = 4$$

$$a = \underline{\underline{2\sqrt{3}}}$$